

# Segregation and Stability of Binary Granular Mixtures

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We measure stability of two-dimensional granular mixtures in a rotating drum and relate grain configurations to stability. For our system, the smaller but smoother grains cluster near the center of the drum, while the larger, rougher grains remain near the outer edge. One consequence of the size segregation is that the smaller grains heavily influence the stability of the heap. We find that the maximum angle of stability is a non-linear function of composition, changing particularly rapidly when small grains are first added to a homogeneous pile of large grains. We conclude that the grain configuration within the central portion of the heap plays a prominent role in stability.

Granular materials have received much attention due to their important practical applications in fields ranging from geology to the food industry [1]. Despite the simple behavior of individual grains, a granular system can display complex collective behavior. In different regimes of density and motion, granular matter may most mimic a solid, liquid, or gas [2]. Although a good deal of effort has gone into cataloging these phases, there is little understanding of how the microscopic grain configuration leads to specific macroscopic behavior; that is, which features in an arrangement of grains are most important. Observed hysteresis in the transition to crystalline order [3] and in the numbers of contacts between grains [4] demonstrates that the microscopic configuration contains subtle information on the system's history. A more striking illustration of the importance of microstructure comes from sound propagation experiments, where a fractional size change in a single grain can change the total sound transmission through a granular heap by up to 25% [5].

Understanding the connection between configuration and behavior is particularly necessary for moving from experimental studies of homogeneous mixtures to the heterogeneous mixtures that appear in most industrial and other applications. Differences among grain configurations can be more significant in a mixture than in a homogeneous sample. Because the configurations vary more widely, the microscopic arrangements may also have more effect on the overall behavior. Conveniently, the increased variation in heterogeneous systems can also make it easier to address the connection between the microscopic and macroscopic regimes.

Heterogeneous mixtures commonly undergo segregation, a phenomenon of great interest for materials processing [6]. Segregation of binary mixtures also occurs in the laboratory, in both vibration [7, 8] and rotating drum experiments [9, 10, 11, 12, 13, 14, 15], and can be caused by differences in particle size, shape, smoothness, or density. If a particular region of a granular heap plays the major role in some macroscopic behavior, then a mixed heap should behave most like the component that is heavily represented in the region. Here we discuss measurements on binary mixtures in a two-dimensional rotating drum. We describe our apparatus and the segregation patterns we observe. We then combine the segregation observations with measurements of avalanches to probe how different configurations influence stability.

The grains of our mixture, shown in the inset to Figure 1, are composed of  $\frac{1}{8}$ "-diameter ball bearings. We use both sin-

gle spherical bearings and hexagonal shapes created by welding 7 single steel ball bearings together, as described elsewhere [16]. The hexagonal shapes can form near-perfect triangular lattices of the individual spheres, in which the joints between balls are impossible to distinguish, demonstrating that the welding process does not significantly distort the balls.

We use a rotating drum consisting of a  $\frac{1}{8}$ "-thick aluminum sheet with a 13.96"-diameter circle cut out of it. Two  $\frac{1}{2}$ "-thick sheets of Plexiglas sandwich the aluminum sheet and contain the mixture. Confining the grains to a single layer allows us to track visually the exact configuration of the grains during the rotation. In general, only the top few layers of grains flow when a critical angle of stability is reached while the bulk of the mixture remains stable. Extensive three-dimensional segregation experiments and models [12, 13, 14] have reported that binary mixtures will segregate axially into bands. Working in two dimensions eliminates any possibility for axial instability.

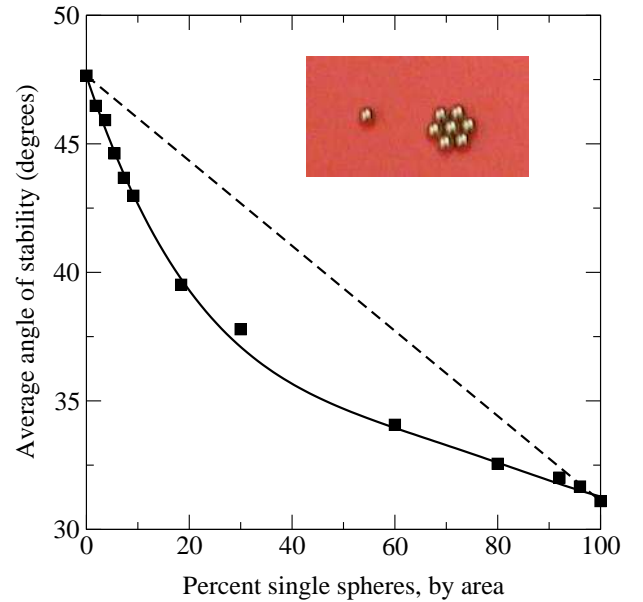


FIG. 1: Maximum stability angles for different pile compositions. The solid line is a guide the eye, and dotted line is the interpolation between the values for homogeneous piles. Inset: sphere and hexagon.

The drum rotates about its center on an axle with rotational speed controlled by a motor. We rotate at  $500 \mu\text{Hz}$ , or one full turn in just over half an hour. This slow speed allows for static limit behavior, characterized by discrete avalanches. The rotation of the drum during an avalanche is about one-third of a degree, for most purposes small enough to be ignored. We also verified experimentally that, within the uncertainty of our measurements, the distribution of avalanches in angle does not change with rotation speed up to  $1 \text{ mHz}$ .

The binary mixture is inserted into the tumbler through an opening in one side of the aluminum sheet. The opening is closed during rotation. Since we are studying the behavior of binary mixtures, the hexagonal shapes must be entered carefully to avoid breakage. As described below, even a small percentage of single spheres in a pile of hexagonal grains noticeably decreases the pile's stability, and broken hexagons have a similar effect. After completing measurements on a pile, we check for broken shapes. If more than 1% of the hexagonal shapes are broken, we discard the data and remeasure that composition.

Another method is to disassemble the tumbler and remove one of the Plexiglas faces. The shapes can then be placed in the aluminum sheet with little risk of breaking. However, if the screws which hold the Plexiglas and the aluminum sheets together are not tightened equally, we find a periodicity in avalanche behavior that reflects the rotation rate of the drum. This arises from slight differences in how uniformly the grains are confined to a single layer. To avoid such bias, we adjust the container until the rotation periodicity is not reflected in our data, and then leave the container undisturbed through the entire set of measurements.

For each run the pile fills about 28% of the container. Once a mixture is loaded into the tumbler, we rotate at  $500 \mu\text{Hz}$  for at least 20 minutes before beginning to record avalanches. This avoids anomalous results arising from the configuration of the grains upon loading. Instead, we consider only avalanches in a “steady state,” where each grain configuration has developed from previous avalanches. We then rotate the drum for one to two hours to obtain about 100 discrete avalanches, filming the entire time with a digital video camera. Avalanches are larger and less frequent for the hexagons than for the spheres, so the filming takes correspondingly longer. The frames taken immediately before and after each avalanche are transferred to a computer. The angle of stability before an avalanche and the angle of repose afterward are calculated from the average slope of the pile's free surface. The pile's packing fraction before and after each avalanche is also computed, as described elsewhere [17].

For many of the measurements we use brass ball bearings for the single balls, which we can distinguish by color from the steel hexagons. Since the brass balls weigh about 10% more than the steel balls, the change in material could affect the segregation and avalanche behavior. For a few mixtures we also performed measurements with all steel balls, and verified that substituting the brass ball bearings does not change the avalanche angles and packing fractions. In Figure 2, we show typical configurations of the mixtures. In the original images, which are included in an appendix, about 70%

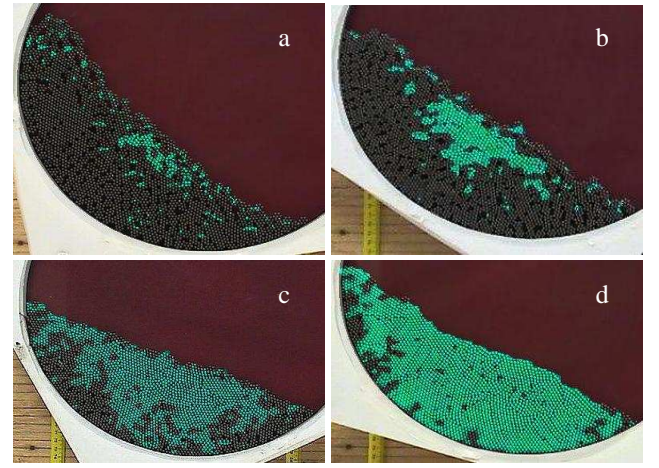


FIG. 2: Progression of segregation for mixture of hexagons (gray) and spheres (green). Shown are mixtures with (a) 7%, (b) 18%, (c) 58%, and (d) 89% single spheres by area.

of the brass balls show unambiguously from their yellowish color. Since light reflected from nearby brass balls or red background can give even the steel balls a slight yellow tint, distinguishing the remaining single brass balls is more difficult. To better convey the configurations in the small images of Figure 2, the brass balls are colored green for visibility. We select those balls with the strongest yellow tint as brass, with the cutoff chosen so that the correct number of balls are identified as brass. A few steel balls with brass neighbors fall in the “brass” group and some isolated brass balls do not. Nonetheless, the bulk of the brass balls are selected accurately and the general patterns shown are correct.

The spheres cluster near the center of the drum, usually slightly below the free surface of the heap. The hexagons tend to inhabit the outer portions of the drum. Several previous studies [11, 12, 13], both numerical and experimental, have also found that size differences lead to radial segregation, with smaller shapes moving to the drum center. Interestingly, other work [9, 10] shows that friction can also cause radial segregation, with rougher shapes moving to the center. The size and roughness characteristics can compete with each other. When a mix of small smooth grains and large rough grains are dropped onto a pile, the grain types form striations within the pile [18]. A much simpler segregation occurs when the smaller grains are also rougher. In this case the larger shape collects at the bottom and the smaller shape at the top of the pile, with a single sharp demarcation between the two regions [18]. Geologists have long been aware of such patterns, using striations to understand how sediment settled out of the flows that created rock structures [19]. For our shapes, the hexagons are clearly rougher, but their large size apparently overwhelms their roughness in the segregation process. We find no striations or fingering, but rather a clustering consistent with pure size segregation. Possibly this behavior would change for large rotation speeds, where the free surface is steepest in the center and the shape with greater repose angle collects there. At our slow speeds, with a near-linear free surface, the reduced influence of roughness is not surprising.

TABLE I: Locations of single spheres for avalanches with angles at least one standard deviation from the mean.

	Highest angles	Lowest angles
Through	0	6
Near	4	5
Far	10	2
Total	14	13

Figure 2 shows the progression of the segregation as the concentration of singles increases. For small percentages (2-8%) the singles, which converge towards the bottom of the laminar layer, group themselves into small clusters. The size and number of these clusters increases with composition. The 7% composition (Figure 2a) shows singles congregating into these distinct clumps of 5 to 10 singles. These clusters all lie within an elongated area running roughly parallel to the free surface.

At 18% spheres (Figure 2b), the small clusters have grown together and the singles have saturated the elongated area near the center of the free surface. With further increase in the fraction of single spheres, this central region grows, spreading outward both along the free edge and perpendicular to it. At the other extreme, in mixtures with primarily single spheres, the few remaining hexagons lodge along the outer edge of the container.

We now turn to the steady-state averages of the maximum angle of stability for our binary mixtures. From previous work [17] we know that hexagons are significantly more stable than single ball bearings. Similar results were reported in [20] for disks and pentagons. As shown in Figure 1, varying the fraction of spheres in the mixture changes the stability of the pile in a decidedly nonlinear manner. For mixtures with a small percentage of spheres, the stability angle decreases nearly three times as quickly as the linear interpolation between the angles for homogeneous piles. At the other extreme, a few hexagons in a pile of spheres have almost no effect on the pile's stability.

In light of the segregation behavior described above, one explanation of the observed angles of stability is that most avalanches are triggered from the central region of the drum, which has a disproportionately large concentration of single spheres. Conversely, a small number of hexagons in a pile of spheres move to the outside of the drum and have almost no effect on the avalanche behavior.

The individual arrangements just before each avalanche make the influence of the central region even more apparent. Figure 3 shows three configurations for the pile with 18% spheres. In the upper image the central clump of single spheres reaches through to the free surface of the pile over a length of several grains. In the middle image the spheres track the free surface, with only a single row of hexagons atop the spheres. In the lower image the single spheres are farther from the boundary, and the few that approach it most closely are not part of the main clump of spheres. Although categorizing a configuration as “through,” “close,” or “far” is subjective, two viewers agreed on over 90% of the images.

For the 18% pile, we examined the images for all

avalanches that occurred at angles at least one standard deviation from the mean, with the results in Table I. A clear distinction emerges despite the subjective nature of the analysis. In all but two of the 13 lowest-angle avalanches, the clump of single spheres extended through to the free surface or came

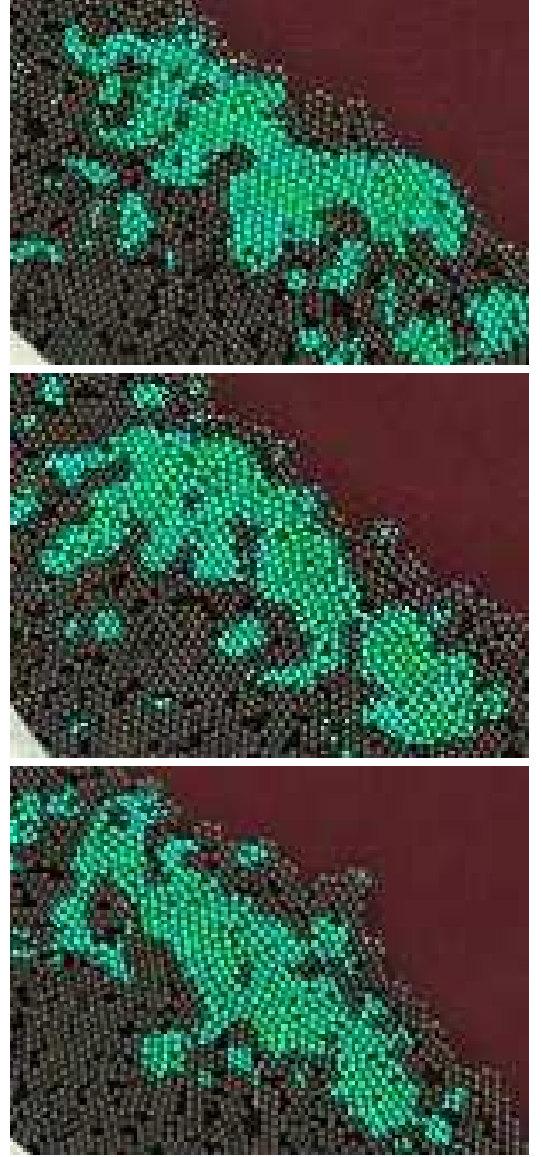


FIG. 3: Examples of the arrangements of single spheres near the free surface of the pile. We classify these as “through” (top), “close” (center), and “far” (bottom) from the surface.

very close. By contrast, for the 14 highest-angle avalanches, the singles never reached the free surface and came close in only four cases. Thus single balls within the top few layers have the greatest impact in triggering avalanches. We find similar trends for other single-hexagon mixtures, although the 18% is the most illuminating. For lower fractions of single spheres the central clump disperses into small clusters and “close” and “through” configurations become very rare. For higher concentrations, the central clump of singles is larger



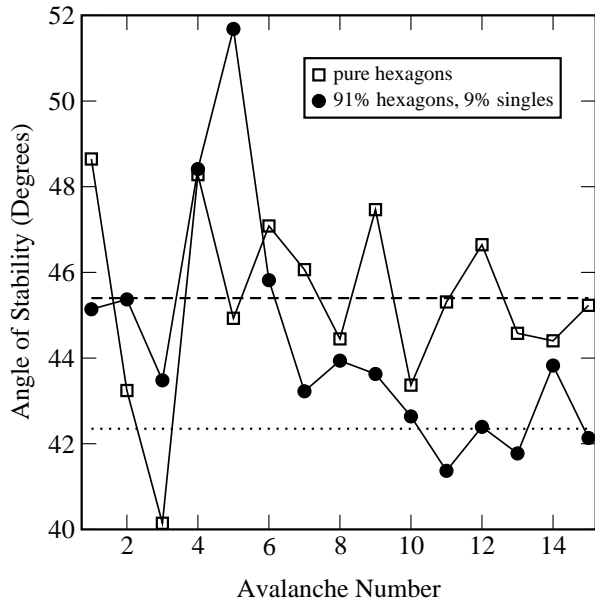


FIG. 4: Avalanche angle for the first several avalanches for homogeneous hexagons (open squares) and a mixture with 9% singles (filled circles). Dashed and dotted lines are averages of the final six avalanches for the pure hexagon and the mixture, respectively.

and nearly always reaches the free surface. In both cases the contrast between the high-angle and low-angle avalanches recedes.

As a further test, we examined stability of configurations far from the typical segregation patterns. Using a mixture of hexagons and spheres, we first loaded the spheres into the drum, so that they clustered along the bottom edge of the drum, and placed the hexagons on top of them. This starts the spheres as far as possible from their eventual position in the center just below the free surface. Therefore, for the first few avalanches the singles are peripheral and we expect to see stability comparable to the initial stability of a homogeneous hexagonal pile. During these first few avalanches, the singles rotate with the drum until they reach the top of the pile and begin participating in the avalanches. Next we expect a transition period in which the less stable spheres are no longer extraneous and the stability angle should decrease. Once the singles have migrated to their usual central position, subsequent avalanches should reflect the usual characteristics of the steady state.

Figure 4 shows this test of initial stability for a mixture

with 9% spheres, as well as a similar measurement of initial avalanches for a homogeneous pile of hexagons. Because of the scatter in individual avalanches, we measure the stability angles for the first 15 avalanches in several trials at each composition. Between trials the drum is emptied and reloaded. The data of Figure 4 are averaged across these trials, avalanche by avalanche. Even for a homogeneous pile, the first few avalanches depend on the packing produced by loading the drum rather than on configurations resulting from previous avalanches, and show wide variations in the maximum angle before the avalanche. The hexagonal pile serves as a control, testing how configurational differences other than segregation affect the stability angles. The first six avalanches of both compositions have average angle of stability similar to that of the homogeneous hexagons. This indicates that, initially, the singles are indeed peripheral both physically and in their effect on the avalanches. After about 10 avalanches, the average stability for the 9% composition has transitioned towards the steady-state average. Visually, this corresponds well to when the single shapes permeate the central area of the mix and start the clumping characteristic of radial segregation. The segregation occurs within about  $\frac{1}{3}$  of a full rotation.

In conclusion, we have measured stability for various mixtures of a small smooth shape and a large rough shape. We find that radial segregation in our system is governed primarily by size difference rather than roughness, with the smaller grains moving to the center of the container. Measuring stability, we find nonlinear behavior characterized by a sharp decrease in stability when a small amount of a low-stability shape is introduced to a homogeneous system. We further show that the nonlinear behavior in stability angle as a function of composition depends on the radial segregation pattern.

We plan to extend this work in several ways. By using faster rotation speeds we can observe dynamical angles of stability. It will be particularly interesting to see whether the rougher hexagonal shapes still segregate to the outside of the container at high speeds. We will also examine in more detail how the segregation occurs, and look at other grain shapes.

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## APPENDIX

Figures 5 and 6 include the images of Figures 2 and 3 with their original colors.

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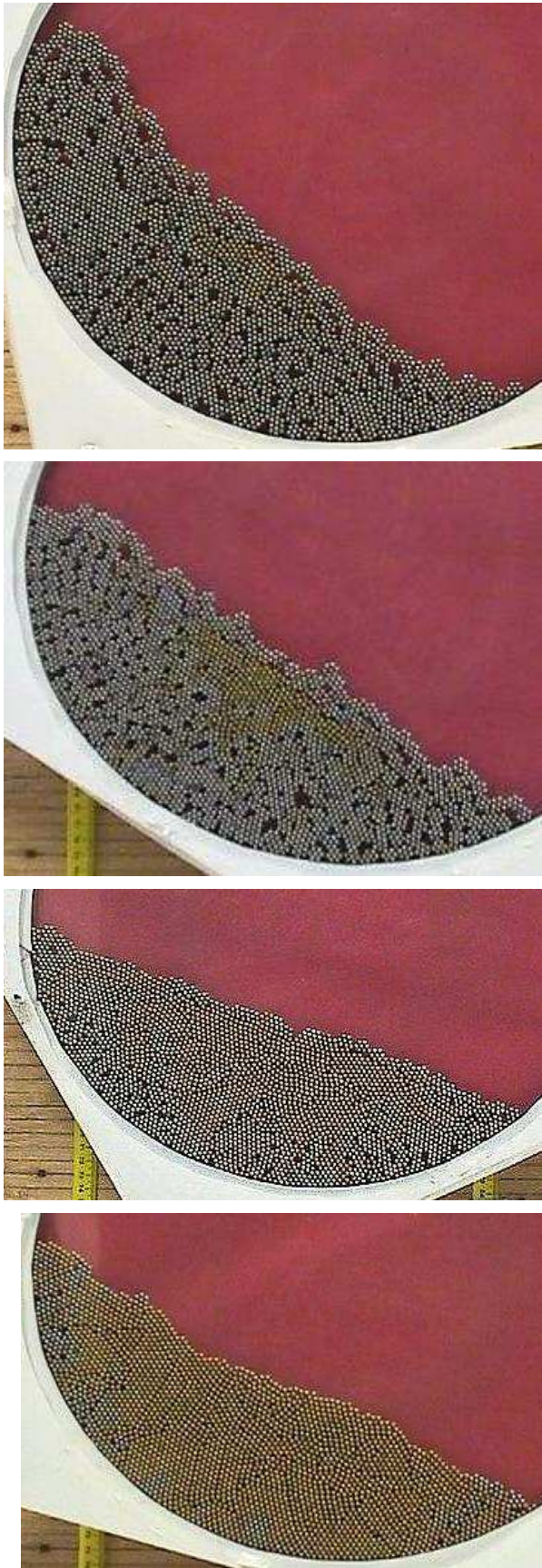


FIG. 5: Images of Figure 2 with original colors.

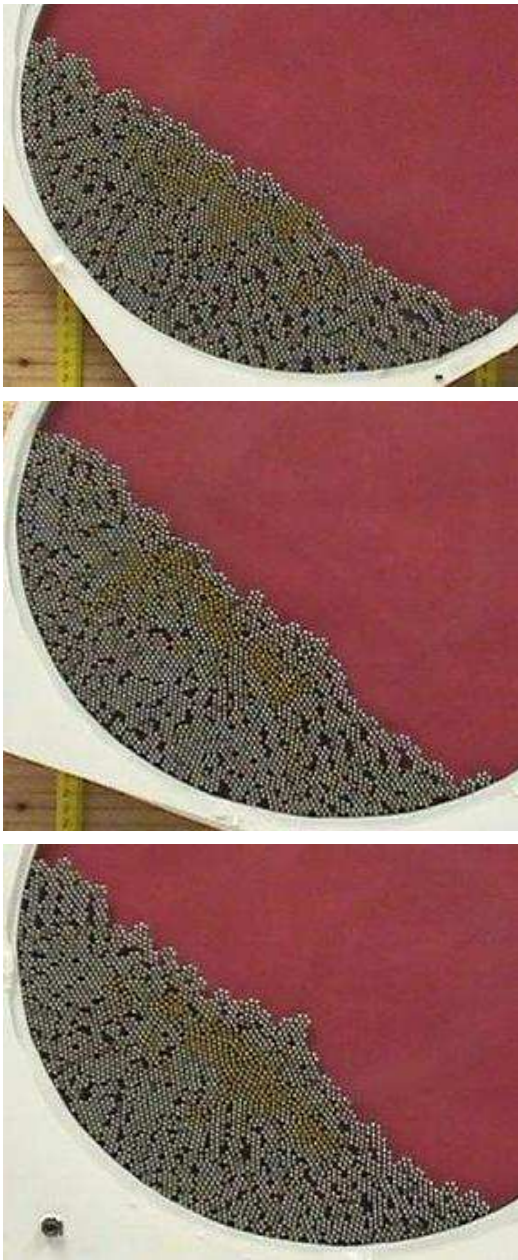


FIG. 6: Images of Figure 3 with original colors.